Your notes

GCSE Maths: revision notes

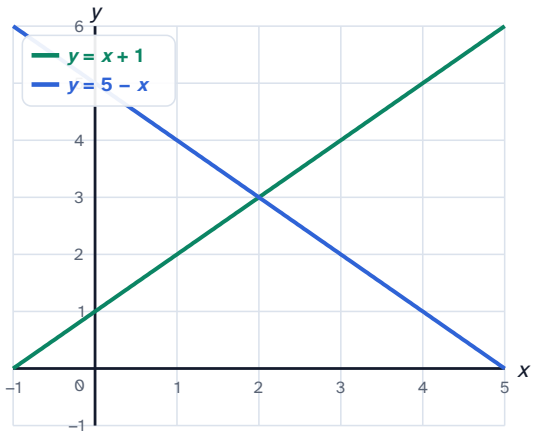
Simultaneous equations

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What they are

- **Simultaneous equations** are two equations with two unknowns. The solution is the pair of values that makes both equations true.
- On a graph, the solution is the point where the two lines cross.



Elimination

- Add or subtract the equations to get rid of one letter. x + y = 10 and x – y = 4: adding gives 2x = 14, so x = 7 and y = 3.
- If no **coefficients** match, multiply first. 3x + 4y = 18 and 5x – 2y = 4: double the second to get 10x – 4y = 8, then add: 13x = 26, so x = 2 and y = 3.
- Same signs: subtract. Different signs: add. Always substitute back to find the other letter, then check in the other equation.

Substitution

- If one equation says y = ..., put it into the other. y = 2x + 1 and 3x + y = 11: 3x + 2x + 1 = 11, so x = 2 and y = 5.
- (Higher) A linear and a quadratic equation: substitute the linear one into the quadratic. y = x + 1 and $x^2 + y^2 = 13$ give $x^2 + (x + 1)^2 = 13$, so $x^2 + x - 6 = 0$, which gives (2, 3) and (–3, –2).

Forming equations from words

- Choose a letter for each unknown, write two equations, then solve. '2 coffees and a tea cost £7; a coffee and a tea cost £4.20' gives 2c + t = 7 and c + t = 4.20, so c = £2.80.

Worked examples

Solve 3x + 4y = 18 and 5x – 2y = 4

3x + 4y = 18 (1)
5x – 2y = 4 (2)

Make the y terms match: multiply equation (2) by 2. **10x – 4y = 8 (3)**

The signs of the y terms are different, so add (1) and (3). The y terms cancel. **13x = 26**

Divide both sides by 13. **x = 2**

Put x = 2 into equation (1). **6 + 4y = 18**

Subtract 6 from both sides, then divide by 4. **4y = 12, so y = 3**

Check in equation (2): 5 × 2 – 2 × 3 = 10 – 6 = 4, which is right. **4** ✓

Answer: x = 2, y = 3

Solve y = 2x + 1 and 3x + y = 11 by substitution

y = 2x + 1 (1)
3x + y = 11 (2)

Equation (1) says what y is, so replace y in equation (2) with 2x + 1. **3x + 2x + 1 = 11**

Collect the x terms. **5x + 1 = 11**

Subtract 1 from both sides, then divide by 5. **5x = 10, so x = 2**

Put x = 2 into equation (1) to find y. **y = 2 × 2 + 1 = 5**

Check in equation (2): 3 × 2 + 5 = 11. **11** ✓

Answer: x = 2, y = 5

Your notes

(Higher) Solve $y = x + 1$ and $x^2 + y^2 = 13$

Substitute $y = x + 1$ into the second equation. **$x^2 + (x + 1)^2 = 13$**

Expand the bracket: $(x + 1)^2 = x^2 + 2x + 1$. **$x^2 + x^2 + 2x + 1 = 13$**

Collect the terms and make one side 0. **$2x^2 + 2x - 12 = 0$**

Divide every term by 2. **$x^2 + x - 6 = 0$**

Factorise: two numbers that multiply to -6 and add to 1 are 3 and -2. **$(x + 3)(x - 2) = 0$**

Solve each bracket. **$x = -3$ or $x = 2$**

Find y for each one using $y = x + 1$. **$x = -3$ gives $y = -2$; $x = 2$ gives $y = 3$**

Answer: $x = 2$, $y = 3$ or $x = -3$, $y = -2$

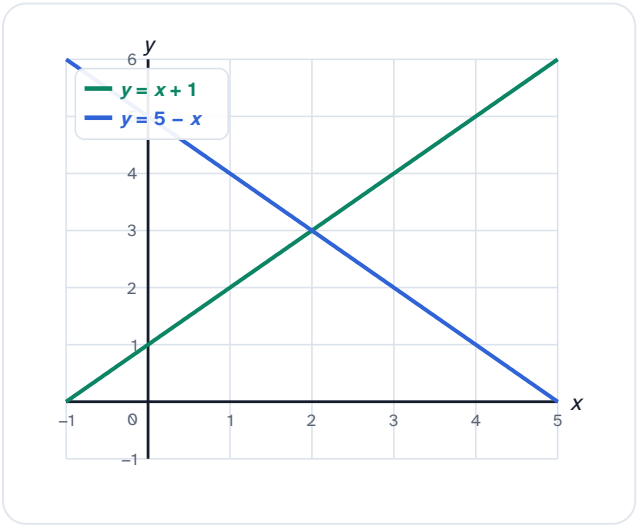
Key terms

Simultaneous equations	Two equations with two unknowns that are true at the same time.
Elimination	Adding or subtracting equations to remove one unknown.
Substitution	Replacing a letter with an expression or value from another equation.
Coefficient	The number in front of a letter.
Point of intersection	The point where two graphs cross, which gives the solution.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. Solve $x + y = 10$ and $x - y = 4$.
2. Solve $2x + y = 11$ and $x + y = 7$.
3. To eliminate y from $3x + 2y = 12$ and $5x - 2y = 4$, what should you do?
4. Solve $y = 2x + 1$ and $3x + y = 11$.
5. What does the solution of two linear simultaneous equations show on a graph?



Your notes

6. Solve $3x + 4y = 18$ and $5x - 2y = 4$.
7. Two numbers add to 20 and have a difference of 6. What are they?
8. 2 coffees and 1 tea cost £7. 1 coffee and 1 tea cost £4.20. How much is a coffee?

Answers

1. x = 7, y = 3
2. x = 4, y = 3
3. Add the equations
4. x = 2, y = 5
5. Where the two lines cross
6. x = 2, y = 3
7. 13 and 7
8. £2.80

What to do next

Practise 12 questions on this topic

Try 6 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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