Your notes

GCSE Maths: revision notes

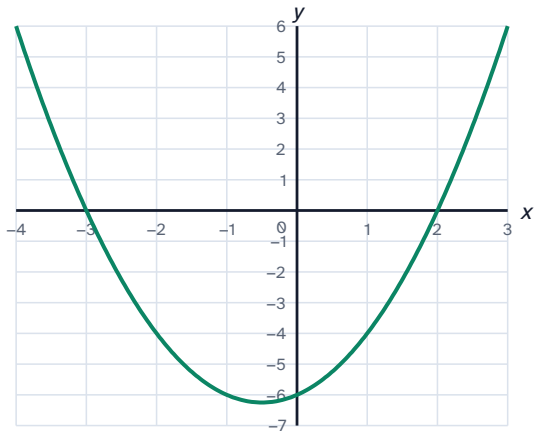
Quadratic equations

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Quadratic equations

- A **quadratic equation** can be written as $ax^2 + bx + c = 0$. It can have two solutions (**roots**), one repeated solution, or no real solutions.
- Always rearrange so one side is 0 before solving.



Solving by factorising

- Factorise, then set each bracket equal to 0: $x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0 \rightarrow x = -2$ or $x = -3$.
- $x^2 = 25$ gives $x = 5$ or $x = -5$. Don't forget the negative **root**.
- $x^2 - 7x = 0 \rightarrow x(x - 7) = 0 \rightarrow x = 0$ or $x = 7$. Never divide both sides by x: you lose the solution $x = 0$.
- (Higher) $2x^2 + 7x + 3 = 0 \rightarrow (2x + 1)(x + 3) = 0 \rightarrow x = -\frac{1}{2}$ or $x = -3$.

The quadratic formula (Higher)

- $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$. Use it when the quadratic won't factorise, and give answers to the accuracy asked.
- $2x^2 - 3x - 7 = 0$: $a = 2$, $b = -3$, $c = -7$, so $x = (3 \pm \sqrt{65}) / 4$, giving $x = 2.77$ or $x = -1.27$ (2 d.p.).
- The value of $b^2 - 4ac$ tells you how many solutions there are: positive gives two, zero gives one, negative gives none.

Completing the square (Higher)

- $x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$. For example, $x^2 + 6x + 2 = (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$.
- The **turning point** of $y = (x + 3)^2 - 7$ is $(-3, -7)$.
- To solve $(x + 3)^2 - 7 = 0$: $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

Worked examples

Solve $x^2 + 5x + 6 = 0$

Find two numbers that multiply to 6 and add to 5. **2 and 3**

Write it in brackets. **$(x + 2)(x + 3) = 0$**

If two things multiply to make 0, one of them is 0. **$x + 2 = 0$ or $x + 3 = 0$**

Answer: $x = -2$ or $x = -3$

(Higher) Solve $2x^2 - 3x - 4 = 0$, giving your answers to 2 decimal places

It doesn't factorise, so use the quadratic formula with $a = 2$, $b = -3$ and $c = -4$. **$x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$**

Work out $b^2 - 4ac$ first. **$(-3)^2 - 4 \times 2 \times (-4) = 9 + 32 = 41$**

Put the numbers into the formula. **$x = (3 \pm \sqrt{41}) \div 4$**

Work out the + answer. **$(3 + 6.4031...) \div 4 = 2.3507...$**

Work out the – answer. **$(3 - 6.4031...) \div 4 = -0.8507...$**

Answer: $x = 2.35$ or $x = -0.85$

Key terms

Quadratic equation	An equation where the highest power of x is 2.
Root	A solution of an equation; where a graph crosses the x-axis.
Quadratic formula	$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$, which solves any quadratic.
Completing the square	Writing a quadratic in the form $(x + p)^2 + q$.
Turning point	The minimum or maximum point of a quadratic graph.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. Solve $x^2 + 5x + 6 = 0$.
2. Solve $x^2 = 25$.
3. Solve $x^2 - 7x = 0$.
4. Solve $(x - 4)(x + 1) = 0$.
5. Solve $x^2 - 9x + 20 = 0$.
6. Why must you not divide both sides of $x^2 = 3x$ by x?
7. Solve $x^2 - 16 = 0$.
8. Solve $2x^2 + 7x + 3 = 0$. (Higher)

Answers

1. $x = -2$ or $x = -3$
2. $x = 5$ or $x = -5$
3. $x = 0$ or $x = 7$
4. $x = 4$ or $x = -1$
5. $x = 4$ or $x = 5$
6. You would lose the solution $x = 0$
7. $x = 4$ or $x = -4$
8. $x = -0.5$ or $x = -3$

What to do next

Practise 12 questions on this topic

Try 7 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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