Your notes

GCSE Maths: revision notes

Iteration

Contents

1. Showing a solution is between two values
2. Iterative formulae
3. Calculator method
4. Key terms
5. Quick questions
6. What to do next

Showing a solution is between two values

- If the value of an expression changes sign between $x = a$ and $x = b$, the equation 'expression = 0' has a solution between a and b.
- For $x^3 + x - 4 = 0$: when $x = 1$ the value is -2; when $x = 2$ it is 6. The sign changes, so there is a solution between 1 and 2.

Iterative formulae

- Rearrange the equation into the form $x = ...$, for example $x^3 - 2x - 5 = 0$ becomes $x^3 = 2x + 5$, so $x = \sqrt[3]{(2x + 5)}$.
- This gives the **iterative formula** $x_{n+1} = \sqrt[3]{(2x_n + 5)}$. Put in the **starting value** x_0 to get x_1, put x_1 in to get x_2, and so on.
- With $x_0 = 2$: $x_1 = 2.0801$, $x_2 = 2.0924$, $x_3 = 2.0942$ (4 d.p.). The values settle towards the solution.

Calculator method

- Type the **starting value** and press =. Then type the formula using the ANS key in place of x_n. Each press of = gives the next value.
- Keep the full calculator values and only round your final answer.
- If the values settle to a fixed number, the **iteration converges**. If they spread out, it does not.

Key terms

Iteration	Repeating a process, using each output as the next input.
Iterative formula	A formula such as $x_{n+1} = \sqrt[3]{(2x_n + 5)}$ that gives the next value from the previous one.
Converge	Settle towards a fixed value.

Change of sign	When a value goes from negative to positive, or positive to negative, showing a solution in between.
Starting value	The first value x_0 used in an iteration.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is iteration?
2. When $x = 1$, $x^3 + x - 4 = -2$. When $x = 2$, it equals 6. What does this show?
3. Using $x_{n+1} = 3 + 1/x_n$ with $x_0 = 1$, what is x_1?
4. Using $x_{n+1} = 3 + 1/x_n$ with $x_0 = 1$, what is x_2?
5. What does it mean if an iteration converges?
6. Which rearrangement of $x^2 - 5x + 2 = 0$ could be used as an iterative formula?
7. What is the value of $x^3 + x - 4$ when $x = 1$?
8. Why should you keep full calculator values when iterating?

Answers

1. Repeating a process, using each output as the next input
2. There is a solution of $x^3 + x - 4 = 0$ between $x = 1$ and $x = 2$
3. 4
4. 3.25
5. The values get closer and closer to a fixed value
6. $x = (x^2 + 2)/5$
7. -2
8. Rounding early can make the final answer inaccurate

What to do next

Practise 12 questions on this topic

Try 5 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

Your notes