Your notes

GCSE Maths: revision notes

Differentiation and turning points

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Start here: what differentiation does

- Differentiation finds the **gradient** of a curve at any point. The result, written dy/dx, is called the **derivative** or **gradient** function.
- A straight line has the same **gradient** everywhere. On a curve the **gradient** changes, and dy/dx tells you what it is for any value of x.

The rule

- If $y = x^n$, then $dy/dx = nx^{n-1}$: multiply by the power, then take one off the power.
- If $y = ax^n$, then $dy/dx = anx^{n-1}$. A number on its own (a constant) differentiates to 0, and $y = kx$ differentiates to k.
- Differentiate each term separately: if $y = x^3 + 4x^2 - 5x + 7$, then $dy/dx = 3x^2 + 8x - 5$.
- Write fractions as powers first: $1/x = x^{-1}$, so its **derivative** is $-x^{-2}$, which is $-1/x^2$.

Gradients at a point

- To find the **gradient** at a point, substitute its x value into dy/dx.
- Example: for $y = x^2 - 3x$ at $x = 4$, $dy/dx = 2x - 3$, so the **gradient** is $2 \times 4 - 3 = 5$.

Turning points

- At a **turning point** (a maximum or a minimum) the **gradient** is zero. Set dy/dx = 0 and solve to find x, then substitute into the equation to find y.
- To decide which it is, differentiate again to get d^2y/dx^2. If it is positive the point is a minimum; if it is negative, a maximum. You can also use the shape of the curve.

Rates of change and motion

- If s is the displacement at time t, then **velocity** v = ds/dt and acceleration a = dv/dt.
- Example: $s = t^3 - 2t$, so $v = 3t^2 - 2$ and $a = 6t$.

Exam tip

- Write the **derivative**, then the equation dy/dx = 0, then its solution: each step earns a mark. Don't forget the y coordinate of a **turning point**.

Common mistakes

- Forgetting that a constant differentiates to 0.
- Taking one off the power before multiplying by it. Always multiply by the original power.
- Stopping at the x value of a **turning point** instead of finding y as well.

Worked examples

Find the coordinates of the turning point of $y = x^2 - 6x + 5$.

Differentiate. **$dy/dx = 2x - 6$**

Set the gradient to zero and solve. **$2x - 6 = 0$, so $x = 3$**

Substitute x = 3 into the equation of the curve. **$y = 9 - 18 + 5 = -4$**

Answer: (3, –4)

Find the gradient of $y = 2x^3 - x$ at the point where $x = 2$.

Differentiate each term. **$dy/dx = 6x^2 - 1$**

Substitute x = 2. **$6 \times 4 - 1 = 23$**

Answer: 23

Key terms

Derivative	The gradient function dy/dx of a curve.
Gradient	How steep a line or curve is at a point.
Turning point	A maximum or minimum point on a curve, where the gradient is zero.
Tangent	A straight line that touches a curve at one point and has the same gradient there.
Stationary point	A point on a curve where dy/dx = 0.
Velocity	The rate of change of displacement with time, ds/dt.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. Differentiate $y = x^4$.
2. What does a constant, such as 7, differentiate to?
3. Differentiate $y = 5x^2$.
4. What is the gradient of a curve at a turning point?
5. Find dy/dx when $y = x^3 + 2x$.
6. Find the gradient of $y = x^2$ at $x = 3$.
7. At a turning point, d^2y/dx^2 is negative. What kind of point is it?
8. The displacement of an object is $s = 4t^2$. What is its velocity v?

Answers

1. $4x^3$
2. 0
3. 10x
4. Zero
5. $3x^2 + 2$
6. 6
7. A maximum
8. 8t

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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