Your notes

GCSE Maths: revision notes

Circle theorems

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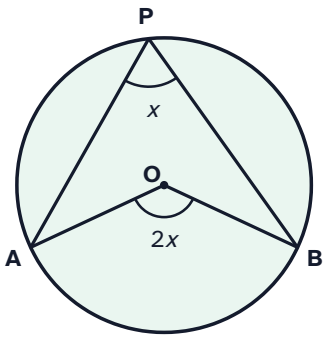
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The parts of a circle

- Radius: from the centre to the edge. Diameter: right across through the centre (two radii). **Circumference**: the edge.
- **Chord**: a straight line joining two points on the circle. Arc: part of the **circumference**. A minor arc is the shorter one, a major arc the longer one.
- Sector: a "pizza slice" between two radii. **Segment**: the region between a **chord** and an arc.
- **Tangent**: a straight line that touches the circle at exactly one point.
- Every radius is the same length, so two radii and a **chord** make an isosceles triangle. Look for these in every question: the two base angles are equal.

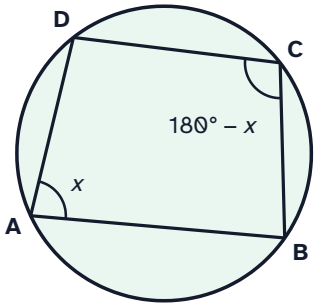
Angles at the centre and circumference

- The angle at the centre is twice the angle at the **circumference** when both stand on the same arc. The shape often looks like an arrowhead (or a "V" inside a bigger "V").
- The angle in a semicircle is 90°: an angle at the **circumference** standing on a diameter is a right angle. (The angle at the centre is 180°, and half of 180° is 90°.)
- Angles in the same **segment** are equal: angles at the **circumference** standing on the same arc. The shape often looks like a bow tie.



Cyclic quadrilaterals

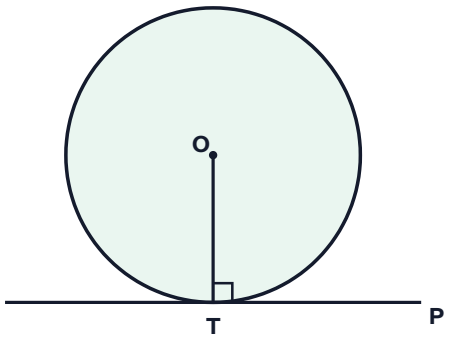
- A **cyclic quadrilateral** has all four corners on a circle. Its opposite angles add up to 180°.
- The centre does not have to be inside it, and the angles next to each other don't have a rule: only opposite pairs add to 180°.
- If one corner of the quadrilateral is the centre of the circle, it is not cyclic: use the angle at the centre rule instead.



Tangents and chords

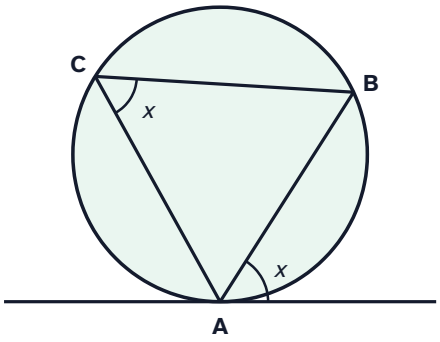
- A **tangent** meets a radius at 90°.
- Two **tangents** drawn to a circle from the same outside point are equal in length. With the two radii they make a kite, which is symmetrical.
- The perpendicular from the centre to a **chord** bisects the **chord** (cuts it into two equal halves). This often leads to Pythagoras: half the **chord**, the distance from the centre and the radius make a right-angled triangle.

Your notes



Alternate segment theorem

- The angle between a **tangent** and a **chord** is equal to the angle in the alternate **segment**: the angle at the circumference on the other side of the chord.
- To spot it: find the **tangent**, follow the **chord** from the point where it touches, and look for a triangle on the far side of the **chord**.



Giving reasons in the exam

- Most circle theorem questions say "give reasons". Write the rule in words for every step, not just the numbers.
- Use the standard wording: "angle at the centre is twice the angle at the **circumference**", "angle in a semicircle is 90°", "angles in the same **segment** are equal", "opposite angles of a **cyclic quadrilateral** add up to 180°", "tangent and radius meet at 90°", "tangents from a point are equal", "perpendicular from the centre bisects a chord", "alternate segment theorem".
- You'll also need the basic angle facts: angles in a triangle add to 180°, on a straight line to 180°, around a point to 360°, in a quadrilateral to 360°, base angles of an isosceles triangle are equal.
- Method: mark every angle you know on the diagram, find isosceles triangles made by radii, then look for the theorem that links an angle you know to the one you want.

Your notes

Proving a theorem (Higher)

- To prove the angle at the centre is twice the angle at the **circumference**, join the point on the **circumference** to the centre. This makes two isosceles triangles (each has two radii).
- If one base angle is x, the other is x too, so the angle at the centre in that triangle is 180 – 2x, and the exterior angle is 2x. Doing the same for the other triangle (with y) gives an angle at the centre of 2x + 2y = 2(x + y), which is twice the angle at the **circumference**, x + y.
- The other theorems can be proved from this one: for example, a **cyclic quadrilateral**'s opposite angles stand on arcs that make 360° at the centre, so they add to half of that, 180°.

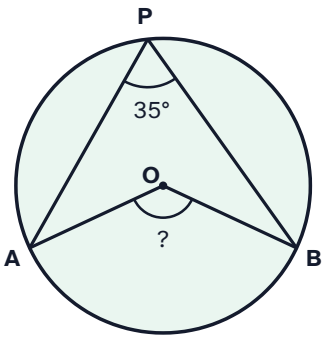
Key terms

Circumference	The edge of a circle.
Cyclic quadrilateral	A quadrilateral with all four corners on a circle.
Tangent	A straight line that touches a circle at one point.
Chord	A straight line joining two points on a circle.
Segment	The region of a circle between a chord and an arc.
Subtended	Formed by an arc or chord at a point, such as the angle an arc makes at the centre.
Alternate segment theorem	The angle between a tangent and a chord equals the angle in the alternate segment.

Quick questions

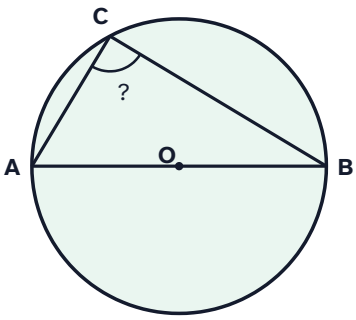
Cover the answers and test yourself: remembering is what makes it stick.

1. The angle at the circumference is 35°. What is the angle at the centre standing on the same arc?

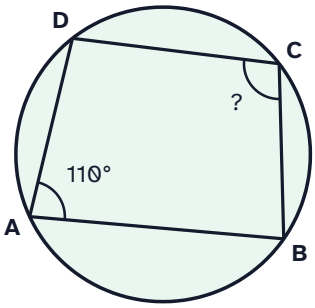


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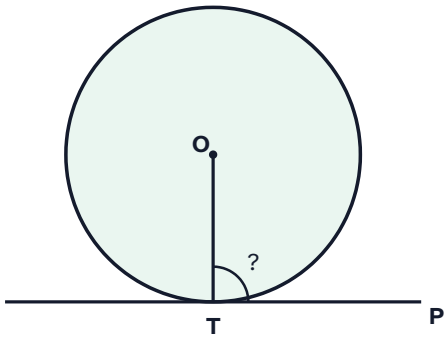
2. What is the angle in a semicircle?



3. One angle of a cyclic quadrilateral is 110°. What is the opposite angle?

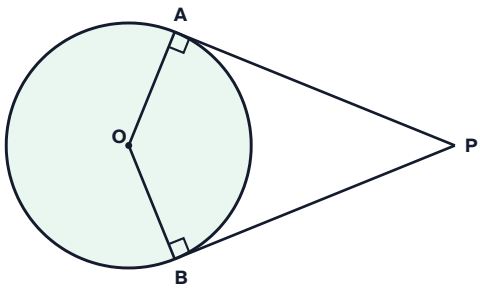


4. What is the angle between a tangent and the radius at the point where they meet?

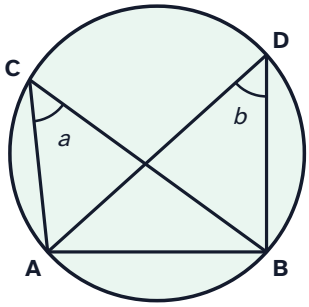


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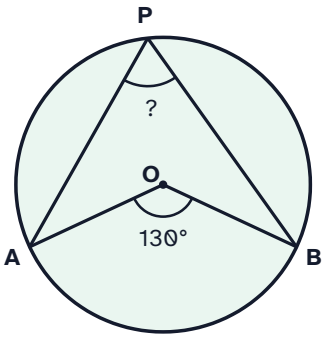
5. Two tangents are drawn to a circle from the same point. What is true about them?



6. What does the perpendicular from the centre to a chord do?
7. What is true about angles in the same segment?



8. The angle at the centre is 130°. What is the angle at the circumference standing on the same arc?



Answers

1. 70°
2. 90°
3. 70°
4. 90°
5. They are equal in length

Your notes

6. It bisects the chord
7. They are equal
8. 65°

What to do next

Practise 22 questions on this topic

Try 8 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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