Your notes

GCSE Maths: revision notes

Equation of a circle and tangents

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Circles centred at the origin

- The **equation of a circle** with centre (0, 0) and **radius** r is $x^2 + y^2 = r^2$.
- $x^2 + y^2 = 25$ has **radius** 5. $x^2 + y^2 = 20$ has **radius** $\sqrt{20} = 2\sqrt{5}$.
- A point is on the circle if its coordinates fit the equation: (3, 4) is on $x^2 + y^2 = 25$ because $9 + 16 = 25$.

Tangents

- A **tangent** touches the circle at one point, and it is **perpendicular** to the **radius** at that point.
- To find a **tangent** at P: find the gradient of the **radius** from the **origin** to P, find the perpendicular gradient (the negative reciprocal), then use $y = mx + c$ through P.
- At (2, 4) on $x^2 + y^2 = 20$: the **radius** has gradient $4 \div 2 = 2$, so the **tangent** has gradient $-1/2$. $4 = -\frac{1}{2} \times 2 + c$ gives $c = 5$, so the **tangent** is $y = -\frac{1}{2}x + 5$.

Lines meeting circles

- Substitute the equation of the line into the equation of the circle and solve the quadratic.
- Two solutions: the line crosses the circle twice. One solution: the line is a **tangent**. No solutions: the line misses the circle.

Key terms

Equation of a circle	$x^2 + y^2 = r^2$, for a circle with centre (0, 0) and radius r.
Radius	The distance from the centre of a circle to its edge.
Tangent	A straight line that touches a circle at exactly one point.
Perpendicular	At right angles to.

Negative reciprocal	The gradient of a perpendicular line: flip the fraction and change the sign.
Origin	The point (0, 0).

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the radius of $x^2 + y^2 = 25$?
2. What is the equation of a circle with centre (0, 0) and radius 3?
3. Does (3, 4) lie on $x^2 + y^2 = 25$?
4. What is the radius of $x^2 + y^2 = 49$?
5. At what angle does a tangent meet the radius?
6. The radius to a point has gradient 2. What is the gradient of the tangent at that point?
7. What is the gradient of the radius from (0, 0) to (3, 4)?
8. What is the gradient of the tangent to $x^2 + y^2 = 25$ at (3, 4)?

Answers

1. 5
2. $x^2 + y^2 = 9$
3. Yes, because 9 + 16 = 25
4. 7
5. 90°
6. –1/2
7. 4/3
8. –3/4

What to do next

Practise 12 questions on this topic

Try 5 exam questions, with mark schemes

Read the lesson, one step at a time

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