Your notes

GCSE Maths: revision notes

Algebraic proof

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Writing numbers in algebra

- If n is an **integer**: 2n is always even, and 2n + 1 is always odd.
- **Consecutive integers**: n, n + 1, n + 2. **Consecutive** even numbers: 2n, 2n + 2. Consecutive odd numbers: 2n + 1, 2n + 3.
- A multiple of 3 can be written 3n. A square number can be written n^2.

Writing a proof

- Write the numbers in algebra, form the expression, then simplify it into a form that shows the result. To show something is a multiple of 4, get it into the form 4(...).
- The sum of three **consecutive integers** is n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1), which is always a multiple of 3.
- $(2n + 1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1$, so the square of an odd number is always 1 more than a multiple of 4.

Identities and counterexamples

- To prove an **identity** (≡), expand and simplify one side until it matches the other.
- One **counterexample** is enough to show a statement is false. 'n^2 + n + 41 is always prime' is false: when n = 40, it equals 1681 = 41 × 41.
- Checking some examples is not a **proof**. A **proof** must work for every value.

Key terms

Proof	An argument that shows a statement is always true.
Counterexample	One example that shows a statement is false.
Identity	A statement that is true for all values, shown with ≡.
Consecutive	Following one after another, such as n and n + 1.
Integer	A whole number: positive, negative or zero.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. If n is an integer, which expression is always even?
2. If n is an integer, which expression is always odd?
3. Which expressions are three consecutive integers?
4. Simplify n + (n + 1) + (n + 2).
5. Why is 3n + 3 always a multiple of 3?
6. Expand $(2n + 1)^2$.
7. How many counterexamples do you need to show a statement is false?
8. Is checking five examples enough to prove a statement?

Answers

1. 2n
2. 2n + 1
3. n, n + 1, n + 2
4. 3n + 3
5. It equals 3(n + 1)
6. $4n^2 + 4n + 1$
7. One
8. No, a proof must work for every value

What to do next

Practise 12 questions on this topic

Try 6 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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