Your notes

Level 2 Further Maths: revision notes

3D problems with Pythagoras and trigonometry

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Pythagoras in 3D

- The diagonal of a cuboid with sides a, b and c is $\sqrt{(a^2 + b^2 + c^2)}$. A 3 × 4 × 12 cuboid has diagonal $\sqrt{(9 + 16 + 144)} = \sqrt{169} = 13$.
- The **space diagonal** of a cube with side s is $s\sqrt{3}$.
- Find lengths step by step: first the diagonal of the base, then use it with the height.

Angles in 3D

- The angle between a line and a **plane** is the angle between the line and its **projection** (shadow) on the **plane**.
- Cuboid with base 3 × 4 and height 12: the base diagonal is 5, so the angle between the **space diagonal** and the base satisfies $\tan \theta = 12 \div 5$, giving $\theta = 67.4°$.
- Pyramids: for a square-based pyramid with base side 6 and height 4 (apex above the centre), the half-diagonal is $3\sqrt{2}$, so each sloping edge is $\sqrt{(18 + 16)} = \sqrt{34}$.
- The angle between two **planes** is found using lines in each **plane** that are perpendicular to where the **planes** meet.

Non-right-angled triangles

- **Sine rule**: $a/\sin A = b/\sin B = c/\sin C$. Use it with a pair of opposite sides and angles.
- **Cosine rule**: $a^2 = b^2 + c^2 - 2bc \cos A$. Use it with two sides and the angle between them, or three sides.
- Area = $\frac{1}{2}ab \sin C$. For sides 6 and 8 with an **included angle** of 30°: $\frac{1}{2} \times 6 \times 8 \times \sin 30° = 12$.

Key terms

Space diagonal	A diagonal passing through the inside of a 3D shape.
Projection	The shadow of a line on a plane.
Plane	A flat surface.
Sine rule	a/sin A = b/sin B = c/sin C.
Cosine rule	$a^2 = b^2 + c^2 - 2bc \cos A$.
Included angle	The angle between two given sides.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the length of the diagonal of a 3 × 4 × 12 cuboid?
2. What is the formula for the diagonal of a cuboid?
3. What is ½ × 6 × 8 × sin 30°?
4. When is the cosine rule used to find a side?
5. To find the angle between a line and a plane, what do you use first?
6. What is the diagonal of a 3 by 4 rectangle?
7. A cuboid has base 3 × 4 and height 12. What is the angle between the space diagonal and the base, to 1 decimal place?
8. What is the sine rule?

Answers

1. 13
2. $\sqrt{(a^2 + b^2 + c^2)}$
3. 12
4. When two sides and the included angle are known
5. The projection of the line onto the plane
6. 5
7. 67.4°
8. a/sin A = b/sin B = c/sin C

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

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