Level 2 Further Maths: revision notes

Tangents and normals

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Tangents

- The **tangent to a curve** at a point has the same gradient as the curve there.
- Method: find the y-coordinate, find dy/dx at the point, then use $y - y_1 = m(x - x_1)$.
- Example: $y = x^2 - 3x + 4$ at $x = 2$. $y = 4 - 6 + 4 = 2$. $dy/dx = 2x - 3 = 1$. The tangent is $y - 2 = 1(x - 2)$, so $y = x$.
- A tangent with gradient 0 is horizontal.

Normals

- The **normal** is perpendicular to the tangent at the same point, so its gradient is $-1 \div$ (tangent gradient).
- For $y = x^2 - 3x + 4$ at (2, 2), the **normal** has gradient -1, so $y - 2 = -(x - 2)$, giving $y = -x + 4$.
- For $y = x^3$ at (1, 1): the tangent gradient is 3, so the tangent is $y = 3x - 2$. The **normal** has gradient $-1/3$: $y - 1 = -(1/3)(x - 1)$, giving $x + 3y = 4$.

Common mistakes

- Use the gradient at the point, not the gradient function itself.
- Substitute into the original equation to find y, not into dy/dx.
- Where lines cross an axis, set $x = 0$ or $y = 0$.

Key terms

Tangent to a curve	A straight line touching the curve with the same gradient at that point.
Normal	A line perpendicular to the tangent at the same point.
Point of contact	The point where a tangent touches a curve.
Perpendicular gradient	The negative reciprocal of a gradient.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the gradient of the tangent to a curve at a point?
2. The tangent has gradient 4. What is the gradient of the normal?
3. $y = x^2 - 3x + 4$. What is the gradient at $x = 2$?
4. What is the tangent to $y = x^2 - 3x + 4$ at (2, 2)?
5. What is the normal to $y = x^2 - 3x + 4$ at (2, 2)?
6. What is the tangent to $y = x^3$ at (1, 1)?
7. What is the normal to $y = x^3$ at (1, 1)?
8. A tangent has gradient 0. What does it look like?

Answers

1. The value of dy/dx at that point
2. –1/4
3. 1
4. y = x
5. y = –x + 4
6. y = 3x – 2
7. x + 3y = 4
8. A horizontal line

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

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Your notes