Your notes

Level 2 Further Maths: revision notes

Surds and indices

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Laws of indices

- $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$.
- $a^0 = 1$, and a negative power means a **reciprocal**: $a^{-n} = 1/a^n$.
- A fractional power means a root: a^(1/n) is the nth root of a, and a^(m/n) = (nth root of a)^m. Take the root first to keep numbers small.
- Examples: 8^(2/3) = $(\sqrt[3]{8})^2 = 2^2 = 4$. 27^(–1/3) = $1/\sqrt[3]{27} = 1/3$. (4/9)^(–1/2) = (9/4)^(1/2) = 3/2.

Solving index equations

- Write both sides with the same base, then compare powers. $2^x = 32 = 2^5$, so x = 5.
- $9^x = 27$ means $(3^2)^x = 3^3$, so 2x = 3 and x = 3/2.

Simplifying surds

- A **surd** is a root that can't be simplified to a whole number, such as √2.
- √(ab) = √a × √b and √(a/b) = √a ÷ √b. Look for square factors: √72 = √36 × √2 = 6√2.
- Add **like surds**: √12 + √27 = 2√3 + 3√3 = 5√3.
- Expand brackets as normal, remembering √a × √a = a: $(1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + 2\sqrt{2}$, and $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$.

Rationalising the denominator

- To remove √a from the denominator, multiply top and bottom by √a: $6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$.
- For a denominator a + √b, multiply top and bottom by a – √b (and the other way round), using the difference of two squares.
- Example: $4/(3 - \sqrt{5}) = 4(3 + \sqrt{5})/((3 - \sqrt{5})(3 + \sqrt{5})) = 4(3 + \sqrt{5})/(9 - 5) = 3 + \sqrt{5}$.

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Key terms

Index	The power a number is raised to.
Reciprocal	One divided by a number.
Fractional index	A power such as 1/n, meaning the nth root.
Surd	A root that can't be simplified to a whole number.
Rationalising the denominator	Rewriting a fraction so there is no surd in the denominator.
Like surds	Surds with the same number under the root, which can be added.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is 8^(2/3)?
2. What is 27^(–1/3)?
3. Simplify √72.
4. Simplify √12 + √27.
5. Simplify $5^3 \times 5^4$.
6. Rationalise 6/√3.
7. Expand (2 + √3)(2 – √3).
8. Solve $9^x = 27$.

Answers

1. 4
2. 1/3
3. 6√2
4. 5√3
5. 5^7
6. 2√3
7. 1
8. x = 3/2

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

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