Your notes

Level 2 Further Maths: revision notes

Stationary points

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Finding stationary points

- A **stationary point** is where the gradient is zero: $dy/dx = 0$.
- Method: differentiate, set $dy/dx = 0$, solve for x, then substitute into y to find the coordinates.
- $y = x^2 - 4x + 1$: $dy/dx = 2x - 4 = 0$, so $x = 2$ and $y = -3$. The **stationary point** is $(2, -3)$.

Maximum or minimum?

- Check the gradient either side. Positive, then 0, then negative means a maximum. Negative, then 0, then positive means a minimum.
- Or use the **second derivative**, d^2y/dx^2: positive means a minimum, negative means a maximum.
- A **stationary point** can also be a **point of inflection**, where the curve flattens but keeps going the same way (such as $y = x^3$ at the origin).

Worked example

- $y = x^3 - 3x^2 - 9x + 5$. $dy/dx = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$, so $x = 3$ or $x = -1$.
- At $x = 3$: $y = 27 - 27 - 27 + 5 = -22$. At $x = -1$: $y = -1 - 3 + 9 + 5 = 10$.
- $d^2y/dx^2 = 6x - 6$. At $x = 3$ it is 12 (positive), so $(3, -22)$ is a minimum. At $x = -1$ it is -12 (negative), so $(-1, 10)$ is a maximum.

Increasing and decreasing

- A function is increasing where $dy/dx > 0$. $y = x^2 - 4x + 1$ is increasing for $x > 2$, and decreasing for $x < 2$.

Key terms

Stationary point	A point where the gradient is zero.

Maximum point	A stationary point where the curve changes from going up to going down.
Minimum point	A stationary point where the curve changes from going down to going up.
Point of inflection	A stationary point where the curve doesn't change direction.
Second derivative	The result of differentiating dy/dx again, written d^2y/dx^2.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the gradient at a stationary point?
2. What is the stationary point of $y = x^2 - 4x + 1$?
3. Is the stationary point of $y = x^2 - 4x + 1$ a maximum or minimum?
4. What is the stationary point of $y = 6x - x^2$?
5. The gradient goes positive, zero, then negative. What type of point is it?
6. The gradient goes negative, zero, then positive. What type of point is it?
7. If d^2y/dx^2 is positive at a stationary point, what type is it?
8. What are the x-coordinates of the stationary points of $y = x^3 - 3x^2 - 9x + 5$?

Answers

1. 0
2. (2, –3)
3. Minimum
4. (3, 9)
5. Maximum
6. Minimum
7. Minimum
8. x = 3 and x = –1

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.