Your notes

Level 2 Further Maths: revision notes

Sequences and limiting values

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Linear and quadratic sequences

- A linear sequence has a **common difference** d, and **nth term** dn + (first term – d). 5, 8, 11, 14 has **nth term** 3n + 2.
- A **quadratic sequence** has a constant **second difference**. For $an^2 + bn + c$, the **second difference** is 2a.
- Method: find a from the **second difference**, subtract an^2 from each term, then find the linear **nth term** of what is left.
- Example: 3, 9, 19, 33. The **second difference** is 4, so a = 2. Subtracting $2n^2$ (2, 8, 18, 32) leaves 1, 1, 1, 1. The **nth term** is $2n^2 + 1$.

Using the nth term

- Find a term by substituting: the 10th term of $n^2 + 2n$ is 100 + 20 = 120.
- To check whether a number is in the sequence, set the **nth term** equal to it and see if n is a positive whole number. 3n + 2 = 100 gives n = 98/3, so 100 isn't a term.
- To find which term has a value, solve an equation: $n^2 – 3n + 7 = 25$ gives $n^2 – 3n – 18 = 0$, (n – 6)(n + 3) = 0, so n = 6.

Limiting values

- As n gets very large ($n \to \infty$), some sequences get closer and closer to a **limiting value**.
- Divide the top and bottom by n: (2n + 1)/(n + 3) = (2 + 1/n)/(1 + 3/n). As $n \to \infty$, $1/n \to 0$, so the limit is 2.
- The limit is the ratio of the n coefficients: (3n – 1)/(6n + 5) → 3/6 = 1/2.

Key terms

nth term	A formula giving any term of a sequence from its position n.

Common difference	The fixed amount added each time in a linear sequence.
Second difference	The difference between consecutive first differences.
Quadratic sequence	A sequence with a constant second difference.
Limiting value	The value a sequence approaches as n becomes very large.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the nth term of 5, 8, 11, 14?
2. For $an^2 + bn + c$, what is the second difference?
3. What is the nth term of 3, 9, 19, 33?
4. What is the limiting value of $(2n + 1)/(n + 3)$?
5. What is the limiting value of $(3n - 1)/(6n + 5)$?
6. What is the 10th term of $n^2 + 2n$?
7. Is 100 a term of the sequence $3n + 2$?
8. What is the limiting value of $1/n$?

Answers

1. $3n + 2$
2. $2a$
3. $2n^2 + 1$
4. 2
5. 1/2
6. 120
7. No
8. 0

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.