Your notes

Level 2 Further Maths: revision notes

# The factor theorem

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## The factor theorem

- If $f(a) = 0$, then $(x – a)$ is a factor of $f(x)$. And if $(x – a)$ is a factor, then $f(a) = 0$.
- For a factor $(ax – b)$, substitute $x = b/a$. To show $(2x – 1)$ is a factor, show $f(1/2) = 0$.
- Try small values (±1, ±2, ±3...) that divide the constant term.

## Factorising a cubic

- $f(x) = x^3 – 6x^2 + 11x – 6$. $f(1) = 1 – 6 + 11 – 6 = 0$, so $(x – 1)$ is a factor.
- Divide, or compare coefficients, to find the **quadratic factor**: $x^3 – 6x^2 + 11x – 6 = (x – 1)(x^2 – 5x + 6)$.
- Factorise the quadratic: $(x – 1)(x – 2)(x – 3)$. So $f(x) = 0$ has solutions $x = 1$, 2 and 3.
- A **cubic** has at most three real **roots**.

## Finding unknown coefficients

- If $(x – 2)$ is a factor of $x^3 + ax^2 – 4x + 4$, then $f(2) = 0$: $8 + 4a – 8 + 4 = 0$, so $4a = –4$ and $a = –1$.
- With two unknowns, use two facts to make simultaneous equations.

## Key terms

| | |
|---|---|
| **Factor theorem** | If $f(a) = 0$, then $(x – a)$ is a factor of $f(x)$. |
| **Cubic** | A polynomial with highest power $x^3$. |
| **Root** | A value of $x$ that makes $f(x) = 0$. |
| **Polynomial** | An expression with whole-number powers of $x$. |
| **Quadratic factor** | The quadratic left after taking a linear factor out of a cubic. |

## Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. $f(3) = 0$. Which must be a factor of $f(x)$?
2. $f(x) = x^3 - 6x^2 + 11x - 6$. What is $f(1)$?
3. Which is a factor of $x^3 - 2x^2 - 5x + 6$?
4. Solve $(x - 1)(x + 2)(x - 3) = 0$.
5. $(x - 2)$ is a factor of $x^3 + ax^2 - 4x + 4$. What is $a$?
6. To show $(2x - 1)$ is a factor, which value do you substitute?
7. $x^3 - 6x^2 + 11x - 6 = (x - 1) \times$ what?
8. Is $(x + 1)$ a factor of $x^3 + 1$?

### Answers

1. $(x - 3)$
2. $0$
3. $(x - 1)$
4. $x = 1$, $x = -2$ or $x = 3$
5. $-1$
6. $x = 1/2$
7. $x^2 - 5x + 6$
8. Yes

## What to do next

**Practise 10 questions on this topic**

**Try 3 exam questions, with mark schemes**

**Read the lesson, one step at a time**

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