Level 2 Further Maths: revision notes

Equations of circles

Contents

1. Equations of circles
2. Completing the square
3. Tangents and circle facts
4. Key terms
5. Quick questions
6. What to do next

Equations of circles

- A circle with **centre** (0, 0) and **radius** r has equation $x^2 + y^2 = r^2$.
- A circle with **centre** (a, b) and **radius** r has equation $(x - a)^2 + (y - b)^2 = r^2$. For $(x - 2)^2 + (y + 5)^2 = 49$, the **centre** is (2, –5) and the radius is 7.
- Watch the signs: (y + 5) means b = –5.

Completing the square

- If the equation is expanded, complete the square for x and for y.
- $x^2 + y^2 - 6x + 4y - 12 = 0$ becomes $(x - 3)^2 - 9 + (y + 2)^2 - 4 - 12 = 0$, so $(x - 3)^2 + (y + 2)^2 = 25$. The **centre** is (3, –2) and the **radius** is 5.

Tangents and circle facts

- A **tangent** is perpendicular to the **radius** at the point where it touches the circle.
- **Tangent** to $x^2 + y^2 = 25$ at (3, 4): the **radius** has gradient 4/3, so the **tangent** has gradient –3/4. $y - 4 = -(3/4)(x - 3)$, which gives $3x + 4y = 25$.
- The angle in a **semicircle** is 90°, and the perpendicular from the **centre** to a **chord** bisects the chord.
- To test a point, substitute it: for $x^2 + y^2 = 36$, the point (5, 5) gives $50 > 36$, so it is outside the circle.

Key terms

Radius	The distance from the centre of a circle to its edge.
Centre	The point equally distant from every point on the circle.
Tangent	A straight line that touches a circle at one point.
Chord	A straight line joining two points on a circle.
Semicircle	Half a circle.

Your notes

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. What is the equation of the circle with centre (0, 0) and radius 6?
2. What are the centre and radius of $(x - 2)^2 + (y + 5)^2 = 49$?
3. What is the radius of $x^2 + y^2 = 20$?
4. How is a tangent related to the radius at the point of contact?
5. What is the equation of the tangent to $x^2 + y^2 = 25$ at (3, 4)?
6. What is the centre of $x^2 + y^2 - 6x + 4y - 12 = 0$?
7. What is the radius of $x^2 + y^2 - 6x + 4y - 12 = 0$?
8. Is (5, 5) inside, on or outside $x^2 + y^2 = 36$?

Answers

1. $x^2 + y^2 = 36$
2. Centre (2, –5), radius 7
3. $2\sqrt{5}$
4. It is perpendicular
5. $3x + 4y = 25$
6. (3, –2)
7. 5
8. Outside

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

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Your notes