Your notes

Level 2 Further Maths: revision notes

Algebraic proof

Contents

1. Writing numbers algebraically
2. Writing a proof
3. Identities and always-positive expressions
4. Key terms
5. Quick questions
6. What to do next

Writing numbers algebraically

- Let n be an **integer**. An even number is 2n; an odd number is 2n + 1.
- **Consecutive integers**: n, n + 1, n + 2. **Consecutive** even numbers: 2n, 2n + 2. Consecutive odd numbers: 2n + 1, 2n + 3.
- A multiple of k is k × (an **integer**). To prove something is a multiple of 8, write it as 8 × (an **integer** expression).

Writing a proof

- Start from the general expressions, simplify, then factorise to show the result. End with a concluding sentence.
- The sum of three **consecutive integers**: $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$, which is a multiple of 3.
- The difference between the squares of **consecutive** odd numbers: $(2n + 3)^2 - (2n + 1)^2 = 4n^2 + 12n + 9 - 4n^2 - 4n - 1 = 8n + 8 = 8(n + 1)$, a multiple of 8.
- $(n + 3)^2 - (n - 3)^2 = 12n$, which is a multiple of 12.

Identities and always-positive expressions

- ≡ means "identically equal": true for every value. Show $(x + 2)^2 - (x - 2)^2 \equiv 8x$ by expanding the left side.
- To prove an expression is always positive, complete the square: $x^2 - 6x + 10 = (x - 3)^2 + 1$. A square is never negative, so the expression is at least 1.
- n(n + 1) is always even, because one of two **consecutive integers** is even.
- One **counterexample** is enough to disprove a statement. "All prime numbers are odd" is false, because 2 is prime.

Key terms

Proof	A logical argument showing a statement is always true.

Integer	A whole number.
Consecutive	Following one after another.
Identity	An equation true for all values, shown by ≡.
Counterexample	An example that shows a statement is false.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. How do you write any odd number algebraically?
2. How do you write a multiple of 5?
3. What is the sum of n, n + 1 and n + 2?
4. Simplify $(n + 3)^2 - (n - 3)^2$.
5. How do you write two consecutive even numbers?
6. To prove $x^2 - 6x + 10 > 0$ for all x, what do you write it as?
7. What is a counterexample?
8. Why is n(n + 1) always even?

Answers

1. 2n + 1
2. 5n
3. 3n + 3
4. 12n
5. 2n and 2n + 2
6. $(x - 3)^2 + 1$
7. An example that shows a statement is false
8. One of two consecutive integers is even

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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