Your notes

Level 2 Further Maths: revision notes

Algebraic manipulation and the binomial expansion

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Expanding and factorising

- Expand three brackets by multiplying two first: $(x + 1)(x + 2)(x + 3) = (x^2 + 3x + 2)(x + 3) = x^3 + 6x^2 + 11x + 6$.
- Factorise quadratics with $a \neq 1$ by finding two numbers that multiply to ac and add to b. $6x^2 + x - 2$: $ac = -12$, and 4 and -3 work, so $6x^2 + 4x - 3x - 2 = 2x(3x + 2) - (3x + 2) = (3x + 2)(2x - 1)$.
- Always take out common factors first: $x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$.

Algebraic fractions

- Simplify by factorising and cancelling: $(x^2 - 9)/(x^2 + 5x + 6) = (x - 3)(x + 3)/((x + 2)(x + 3)) = (x - 3)/(x + 2)$.
- Add or subtract using a common denominator: $2/(x + 1) + 3/(x - 2) = (2(x - 2) + 3(x + 1))/((x + 1)(x - 2)) = (5x - 1)/((x + 1)(x - 2))$.

Rearranging formulae

- When the **subject** appears twice, collect those terms on one side and factorise.
- $y = (x + 2)/(x - 3)$ gives $y(x - 3) = x + 2$, so $xy - 3y = x + 2$, $xy - x = 3y + 2$, $x(y - 1) = 3y + 2$ and $x = (3y + 2)/(y - 1)$.

The binomial expansion

- The **coefficients** of $(a + b)^n$ come from **Pascal's triangle** (1; 1 1; 1 2 1; 1 3 3 1; 1 4 6 4 1...) or from $nCr = n! \div (r!(n - r)!)$.
- $(a + b)^n = a^n + nC1\ a^{n-1}b + nC2\ a^{n-2}b^2 + ... + b^n$.
- $(2 + x)^3 = 8 + 12x + 6x^2 + x^3$.
- The x^2 term in $(1 + 2x)^5$ is $5C2 \times (2x)^2 = 10 \times 4x^2 = 40x^2$, so the **coefficient** is 40. Remember to raise the whole term, including its number, to the power.

Key terms

Binomial expansion	Expanding $(a + b)^n$ into a sum of terms.
Pascal's triangle	A triangle of numbers giving binomial coefficients.
nCr	The number of ways to choose r items from n, used for binomial coefficients.
Coefficient	The number in front of a term.
Algebraic fraction	A fraction with algebraic expressions in it.
Subject	The letter a formula is rearranged to find.

Quick questions

Cover the answers and test yourself: remembering is what makes it stick.

1. Expand $(x + 1)(x + 2)(x + 3)$.
2. Factorise $6x^2 + x - 2$.
3. Simplify $(x^2 - 9)/(x^2 + 5x + 6)$.
4. What is 5C2?
5. What is the coefficient of x^2 in $(1 + 2x)^5$?
6. Expand $(2 + x)^3$.
7. Which row of Pascal's triangle gives $(a + b)^4$?
8. Make x the subject of $y = (x + 2)/(x - 3)$.

Answers

1. $x^3 + 6x^2 + 11x + 6$
2. $(3x + 2)(2x - 1)$
3. $(x - 3)/(x + 2)$
4. 10
5. 40
6. $8 + 12x + 6x^2 + x^3$
7. 1, 4, 6, 4, 1
8. $x = (3y + 2)/(y - 1)$

What to do next

Practise 10 questions on this topic

Try 3 exam questions, with mark schemes

Read the lesson, one step at a time

Free revision for GCSE and KS3 at lycolearn.co.uk. Some notes are for one exam board only, and say so in brackets.

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